9 Natural Convection

Objective:

Three objectives of this experiment are:

1. To measure temperature profiles in air near a heated vertical plate.
2. To determine the heat transfer coefficient for natural convection heat transfer from a heated vertical plate.
3. To compare the measurements with theory.

System:

A flat vertical plate (Lexan) is heated on one side by boiling water. Heat is conducted through the plate and lost to air on the other side through a natural convection process. Temperatures of the plate at various locations and temperature profiles in the thermal boundary layer are measured using thermocouples.

Theory:

Natural (or free) convection is observed when *density gradients* are present in a fluid acted upon by a *gravitational field*. Our example of this phenomenon is the heated vertical plate exposed to air which, far from the plate, is at rest. Near the plate the air is heated, and the density of this heated air becomes less than that of the ambient air. The gravitational force acting on an infinitesimal control volume of fluid is therefore less, while the buoyancy force, which is due to the hydrostatic pressure gradient, remains the same as far away from the plate, resulting in a net upward force. An inflow of cooler air from outside the *boundary layer* replenishes the rising fluid (entrainment). A steady state situation will be achieved when the temperatures of the plate and the ambient fluid are held constant. In this flow, thermal and velocity boundary layers exist and the velocity and temperature fields are as shown in Figure 1, where δ_T and δ denote the thicknesses of the thermal and velocity boundary layers, respectively. The momentum and energy equations which govern the motion and the heat transfer are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \quad (1)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (2)$$

where:

u, v = x, y components of velocity, respectively
ν = kinematic viscosity of the fluid
α = thermal diffusivity of the fluid
T = fluid temperature

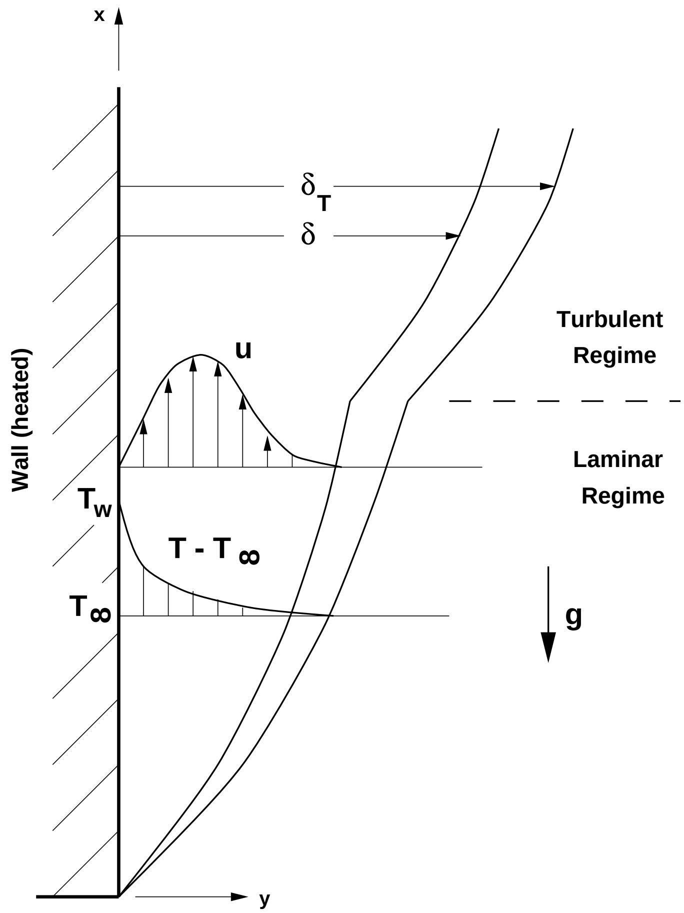

Figure 1: Velocity and temperature profiles in a natural (free) convection boundary layer

T_w = wall surface temperature

T_∞ = fluid temperature far from the wall

g = gravitational acceleration

β = volumetric thermal expansion coefficient $= 1/T$ for ideal gases

The last term in equation (1) is the buoyancy force which drives the flow. The first two terms are inertial (acceleration) terms, the third term is the viscous force (retarding force). The terms on the left-hand side of equation (2) represent energy flowing in and out of a differential control volume, i.e. convection heat transfer and the right-hand side is heat transfer by conduction.

The boundary conditions for the natural convection flow field are:

$$\begin{aligned} u\,,\ v &= 0 \qquad \text{for} \qquad y = 0 \\ T &= T_w \qquad \text{for} \qquad y = 0 \\ u\,,\ v &= 0 \qquad \text{for} \qquad y \to \infty \\ T &= T_\infty \qquad \text{for} \qquad y \to \infty \quad . \end{aligned} \tag{3}$$

Obviously the equations are coupled nonlinear partial differential equations and are difficult to solve exactly. However, for constant fluid properties, solutions exist in the form of *similarity solutions.* These solutions are valid far from any end conditions, and as long as the boundary layer flow is laminar.

Similarity theory for a 2-dimensional natural convection flow transforms the governing coupled nonlinear partial differential equations into coupled nonlinear ordinary differential equations, which reduces the order of independent variables by one (see Incropera/de Witt [20]). It predicts that, for a given Prandtl number, $Pr \equiv \nu/\alpha$, the temperature profiles at various stations x will fall on a single curve, when plotted as

$$\frac{T - T_\infty}{T_w - T_\infty} \quad \text{vs.} \quad \left(\frac{Gr_x}{4}\right)^{1/4} \frac{y}{x} \tag{4}$$

This is the so-called *similarity solution*, which has been obtained numerically and is shown in Figure 2 for a range of Prandtl numbers. In the above equation, the Grashof number (a measure of the buoyancy force times inertia force, divided by the square of the viscous force) is defined as

$$Gr_x \equiv \frac{g\beta(T_w - T_\infty)x^3}{\nu^2} \quad .$$

From Figure 2, the theoretical value of the thermal boundary layer thickness δ_T can be determined. For each station x, it can be taken, for instance, as the location y, where $(T - T_\infty)/(T_w - T_\infty)$ equals 0.01. Finally, and for the sake of completeness, it should be mentioned that, like for temperature, there is a similarity solution for velocity, which is also shown in Figure 2.

Integral Method: It is possible to get an *approximate* value for δ_T using an integral method [30, 33]. For this purpose, the momentum and energy balance is written for a finite segment $\Delta\, x$ of the boundary layer, extending from $y = 0$ to $y = \delta$. For this analysis, it is assumed that both boundary layers (velocity and temperature) have the same thickness. Further, the velocity and temperature profiles within the boundary layer are *assumed* to have the following functional form:

$$\frac{T - T_\infty}{T_w - T_\infty} = \left(1 - \frac{y}{\delta_T}\right)^2 \quad , \; 0 \leq y \leq \delta_T \tag{5}$$

$$\frac{u}{u_{ref}} = \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2 \quad , \; 0 \leq y \leq \delta \tag{6}$$

Of course, these are not the correct profiles (which are given by the similarity solution), but they satisfy the boundary conditions and generally show the right behavior. In equation (6), $u_{ref} = u_{ref}(x)$ is a reference velocity. Finally it is assumed that both δ and u_{ref} follow a power law relationship of the form

$$u_{ref} = Ax^m \; , \quad \delta = B\, x^n \tag{7}$$

Using all the above assumptions, an analytical expression can be derived for the boundary layer thickness:

$$\frac{\delta_T}{x} = 3.93 Pr^{-\frac{1}{2}}(0.952 + Pr)^{\frac{1}{4}} Gr_x^{-\frac{1}{4}} \tag{8}$$

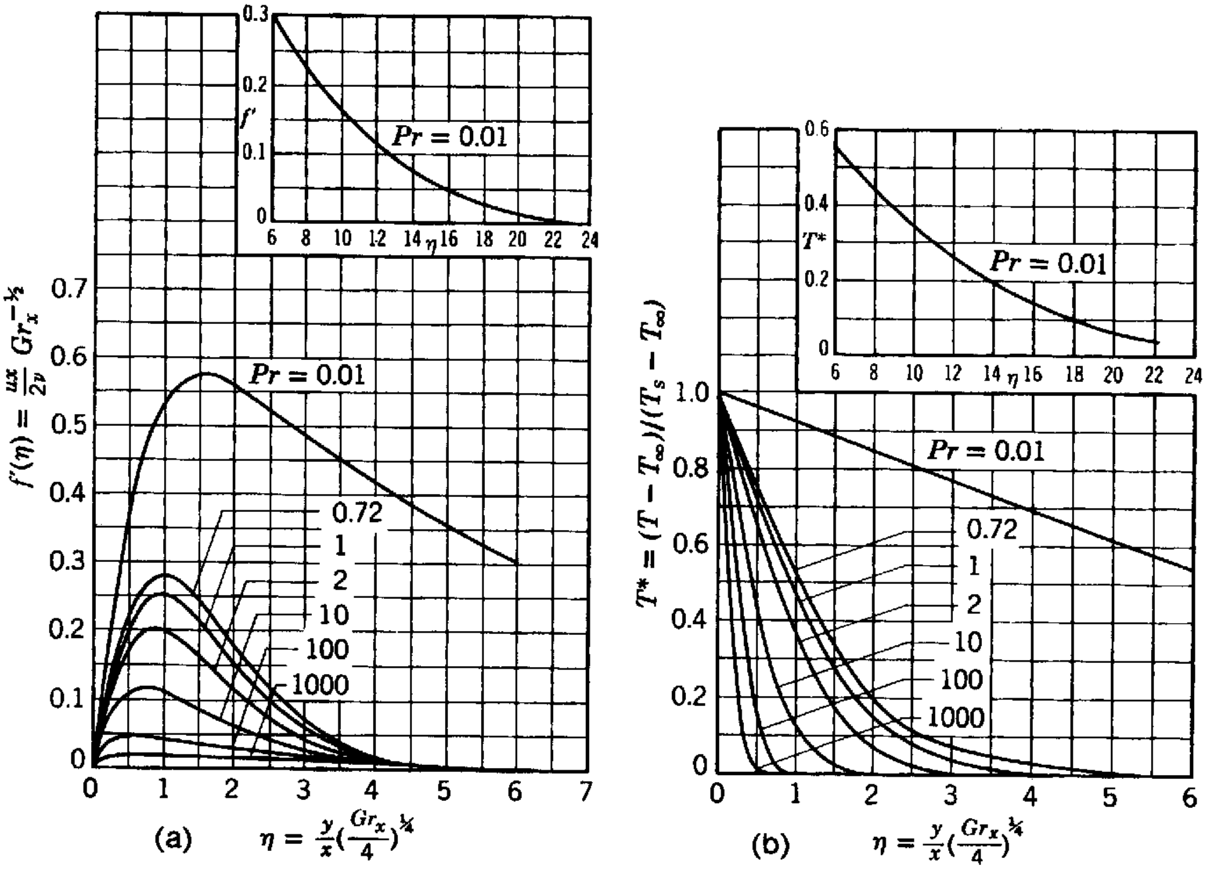

Figure 2: Similarity solution for various Prandtl numbers.

Please keep in mind however, that this expression is only approximate, since many simplifications were used for its derivation.

Heat Transfer: Also of interest is the rate at which heat can be removed from the plate by the air. This can be determined in two different ways. At the surface of the plate, on the air side, the heat flux, q_w is given exactly by:

$$q_w = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{9}$$

Thus, if the thermal contuctivity, k, of the air in contact with the plate is known and if the temperature gradient at the wall can be measured, q_w can be determined. Alternately, since the flux of heat through the air must equal that through the plate we can also write the heat conduction equation for the plate

$$q_w = -k_w \frac{(\Delta T)_w}{L} \tag{10}$$

Here k_w is the thermal conductivity of the plate material, L is the plate thickness and $(\Delta T)_w$ is the difference in temperature of the two surfaces of the plate.

Conventionally, the heat transfer characteristics are expressed in terms of Nusselt number Nu_x which is defined as

$$Nu_x \equiv \frac{h_x x}{k} \tag{11}$$

where the local heat transfer coefficient, h_x, is defined by "Newton's law of cooling"

$$q_w = h_x(T_w - T_\infty) \ . \tag{12}$$

An **analytical** expression for h_x can be obtained from equations (12) and (9), calculating the temperature gradient at the wall from equation (5). Because (5) was merely an assumption, the resulting expression $h_x = 2k \,/\, \delta_T$ is approximate. Inserting (8) for δ_T and using the definition of Nu_x, we obtain an analytical expression for the Nusselt number

$$Nu_x = 0.508 Pr^{\frac{1}{2}}(0.952 + Pr)^{-\frac{1}{4}} Gr_x^{\frac{1}{4}} \tag{13}$$

which is valid only for laminar flow.

An **experimental** expression for h_x can be obtained from equations (12) and (9), calculating the temperature gradient at the wall from your measured data. Using the definition (11) of the Nusselt number, an experimental value for Nu_x can now be calculated as well.

Flow Regime: If the plate is long enough the laminar boundary layer will become unstable and undergo transition to become a fully developed **turbulent** flow. Laboratory measurements show that:

$$\begin{aligned} \text{laminar flow regime:} &\quad = \quad 10^4 < Gr_x Pr < 10^9 \\ \text{turbulent flow regime:} &\quad = \quad Gr_x Pr > 10^9 \end{aligned}$$

In the turbulent flow regime, the temperature and velocity profiles are altered. The result of an integral analysis for the turbulent case is

$$Nu_x = 0.0295\, Gr_x^{\frac{2}{5}}\, Pr^{\frac{7}{15}}\, (1 + 0.494\, Pr^{\frac{2}{3}})^{-\frac{2}{5}} \ . \tag{14}$$

This equation corresponds to equation (13) above.

It should be noted that in the literature, Nusselt number relations like equations (13) and (14) are usually simplified further, or they are adjusted to correlate with experimental data. Typically, such relations are given in the form

$$\overline{Nu} = \overline{Nu}(Gr_x, Pr) \tag{15}$$

where $\overline{Nu}$ is defined as $\overline{Nu} = \overline{h}\, x/k$. Here, $\overline{h}$ is the average value of h_x between the start of the plate and the vertical position x. Typically, in figures presenting such Nusselt number data the laminar and turbulent measurements appear on a single graph.

Your Experiment:

Description of Apparatus

The apparatus is shown schematically in Figure 3, further details are given in Figure 4.
The plate (heat transfer surface) is heated by boiling water in a tank which is insulated on five sides. The water is heated by means of three electrical heaters which are submerged in the tank. The heat transfer surface is protected from air currents by a Plexiglas shield. The location of the thermocouples is shown in Figure 4. The function of each thermocouple is as follows. Thermocouple(s)

1 – 6 measure the Lexan plate temperature on the air side and are used to check for vertical thermal gradients along the plate.

7 measures the ambient temperature.

8 can be moved horizontally and vertically to measure the fluid temperature within the boundary layer.

The leads from the thermocouple switching box are connected to a potentiometer which is used to measure the temperatures from all the thermocouples.

Measurements etc.

1. Measure $T(y)$ at enough x-locations along the plate to obtain a good representation of the natural convection boundary layer. Make sure that in the region adjacent to the plate the data points are spaced closely enough to allow an accurate determination of $\partial T/\partial y$. To be certain, plot the data as you collect it.

2. Measure all other temperatures before and after each profile measurement.

Computations etc.

1. Plot T vs. y for each value of x on a single graph.

2. Determine $(\partial T/\partial y)_{y=0}$ for each profile from your data.

3. Calculate h_x for each profile using the two methods indicated (analytical and experimental).

4. Plot your $T(y)$ data in the similarity form suggested in equation (4) on a single graph.

5. Plot your data (i.e., use your measured h_x) in the form $\log Nu_x$ vs. $\log Gr_x$ and compare with theoretical heat transfer predictions (for example, equation (13)).

Note: All fluid properties are to be evaluated at $T_m = \frac{1}{2}(T_w + T_\infty)$.

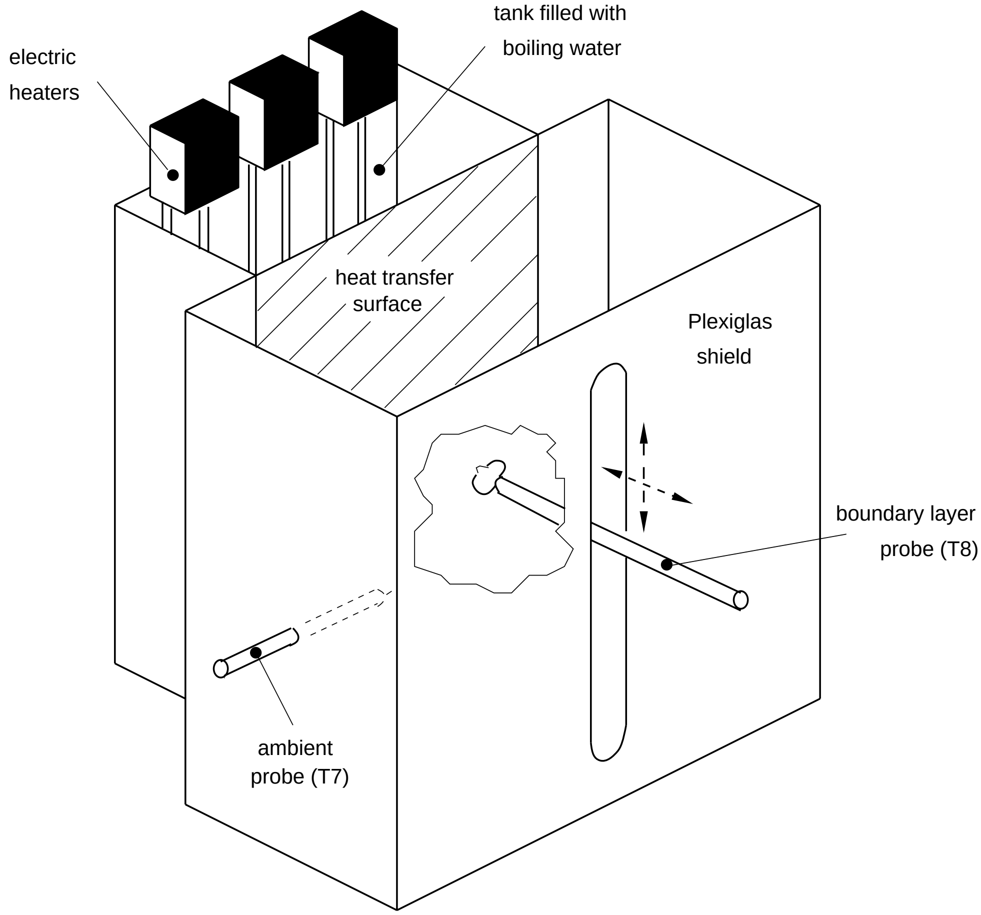

Figure 3: A schematic diagram of the natural convection experiment apparatus

Questions to be addressed in your notebook:

1. Is there "similarity" for the temperature profiles? If not, why not?
2. How does your similarity curve compare with others reported in the literature?
3. Do your Nusselt numbers agree with the theoretical correlation?
4. Is the flow laminar or turbulent? How did you determine this? Could you determine this from the values of the heat transfer coefficient, h_x ?
5. Compare your (experimental) values of h_x with analytical predictions. If there are discrepancies, explain why.
6. Do the boundary conditions in the experiment conform to those in the theory? If there are differences what are they?

References: [5], [8], [14], [17], [20], [30], [33], [40]

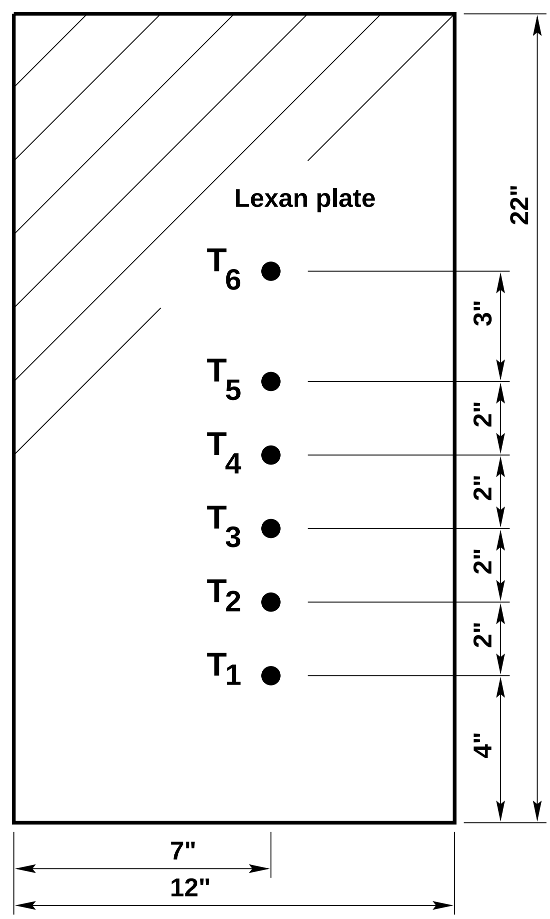

Figure 4: Front view of the heated plate showing the position of the implanted thermocouples